Solutions:

1) a) $v = \langle 1,2,1\rangle$, $w = \langle 3,1,1\rangle$

$$v \times w = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 3 & 1 & 1 \end{vmatrix} = i(2-1) - j(1-3) + k(1-6)$$

$$= i + 2j - 5k = \langle 1, 2, -5\rangle$$

b) $v = \langle 3,0,0\rangle$, $w = \langle -1,0,1\rangle$

$$v \times w = \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ -1 & 0 & 1 \end{vmatrix} = i(0-0) - j(3-0) + k(0-0)$$

$$= -3j = \langle 0,-3,0\rangle$$

2) $(i+j)\times k = i\times k + j \times k = -j + i = \langle 1,-1,0\rangle$

3) $v = \langle a, b, c\rangle$

$$v \times i = \begin{vmatrix} i & j & k \\ a & b & c \\ 1 & 0 & 0 \end{vmatrix} = i(0-0) - j(0-c) + k(0-b)$$

$$= cj - bk = \langle 0, c, -b\rangle$$

$$v \times j = \begin{vmatrix} i & j & k \\ a & b & c \\ 0 & 1 & 0 \end{vmatrix} = i(0-c) - j(0) + k(a)$$

$$= -ci + ak = \langle -c, 0, a\rangle$$

$$v \times k = \begin{vmatrix} i & j & k \\ a & b & c \\ 0 & 0 & 1 \end{vmatrix} = i(b-0) - j(a-0) + k(0)$$

$$= bi - aj = \langle b, -a, 0\rangle$$

⑥ The cross product is <u>NOT</u> associative.

Let $u = \langle 1,0,0 \rangle$, $V = \langle 0,1,0 \rangle$, $w = \langle 1,1,1 \rangle$

Show $(u \times v) \times w \neq u \times (v \times w)$

④ $\|v \times w\| = \|u\|\,\|w\| \sin\theta$

If $v \perp w$, $\theta = \frac{\pi}{2}$, $\Rightarrow \sin\theta = 0$

so $\|v \times w\| = 0$

If $\theta = \pi/4$, $\sin\theta = \sqrt{2}/2$, so $\|v \times w\| = \dfrac{\sqrt{2}\,\|u\|\,\|w\|}{2}$

⑤ <u>Area of parallelogram</u>:

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 2k = \langle 0,0,2 \rangle$$

$$\boxed{\|u \times v\| = 2}$$

<u>Volume of parallelepiped</u>: $|w \cdot (u \times v)|$

$|\langle 1,1,2 \rangle \cdot \langle 0,0,2 \rangle| = 4$

⑥ $$\langle 1,0,0 \rangle \times \langle 0,1,0 \rangle = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = K = \langle 0,0,1 \rangle$$

$$\langle 0,1,0 \rangle \times \langle 1,1,1 \rangle = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = i - 0 \cdot j - K = i - K$$

$\langle 1,0,-1 \rangle$

$$\langle 0,0,1 \rangle \times \langle 1,1,1 \rangle = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = -i - j(-1) + 0K = j - i$$

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$$\langle 1,0,0 \rangle \times \langle 1,0,-1 \rangle = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 0i - j(-1) + 0 = j$$

7) $n \cdot \langle x, y, z \rangle = d \Rightarrow x + y + z = d$

$\Rightarrow 4 - 1 + 1 = d \Rightarrow d = 4$

$$\boxed{x + y + z = 4}$$

8) Parallel $\Rightarrow$ same normal vector

$n = \langle 4, -9, 1 \rangle$

Pass through origin: $\Rightarrow d = 0$

Since eqn is $4x - 9y + z = d$. If $(0,0,0)$ is a solution, then $d = 0$.

$$\boxed{4x - 9y + z = 0}$$

9) Let $x(t) = t$. Then eqns tell us

$y = 1 - x \Rightarrow y(t) = 1 - x(t) = 1 - t$

$2x + y - 3z = 0 \Rightarrow z = \frac{1}{3}(2x + y)$

$\Rightarrow z(t) = \frac{1}{3}\left(2x(t) + y(t)\right) = \frac{1}{3}(2t + 1 - t) = \frac{t}{3} + \frac{1}{3}$

$$\boxed{r(t) = \langle t, 1 - t, t/3 + 1/3 \rangle}$$